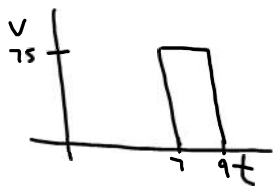
Integral Calculus

The need to calculate instantaneous rates of change led the discoverers of calculus to an investigation of the slopes of tangent lines and, ultimately, to the derivative—to what we call *differential* calculus. But derivatives revealed only half the story. In addition to a calculation method (a "calculus") to describe how functions change at any given instant, they needed a method to describe how those instantaneous changes could accumulate over an interval to produce the function. That is why they also investigated *areas under curves*, which ultimately led to the second main branch of calculus, called *integral* calculus.

Once Newton and Leibniz had the calculus for finding slopes of tangent lines and the calculus for finding areas under curves—two geometric operations that would seem to have nothing at all to do with each other—the challenge for them was to prove the connection that they knew intuitively had to be there. The discovery of this connection (called the Fundamental Theorem of Calculus) brought differential and integral calculus together to become the single most powerful insight mathematicians had ever acquired for understanding how the universe worked.

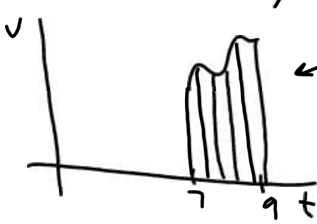
5.1 The Definite Integral

Ex) Train moves 75 mph from 7pm to 9pm.
How far did it go?



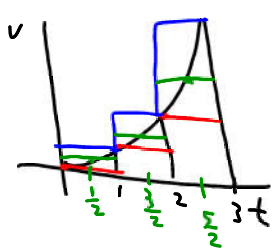
$75 \times 2 = 150$

What if velocity varies?



Add up areas of all strips to approx area.

Ex) Particle starts at $x=0$ at $t=0$.
Moves along x-axis with $v(t)=t^2$.
Where is the particle at $t=3$?



$\boxed{\text{LRAM}}$: $1\cdot(0)^2 + 1\cdot(1)^2 + 1\cdot(2)^2$
$= 0 + 1 + 4 = 5$

RRAM: $1\cdot\underline{(1)}^2 + 1\cdot\underline{(2)}^2 + 1\cdot\underline{(3)}^2$
$= 1 + 4 + 9 = 14$

MRAM: $1\cdot\underline{\left(\tfrac{1}{2}\right)}^2 + 1\cdot\underline{\left(\tfrac{3}{2}\right)}^2 + 1\,\underline{\left(\tfrac{5}{2}\right)}^2$
$= 8\,{}^3/_4$

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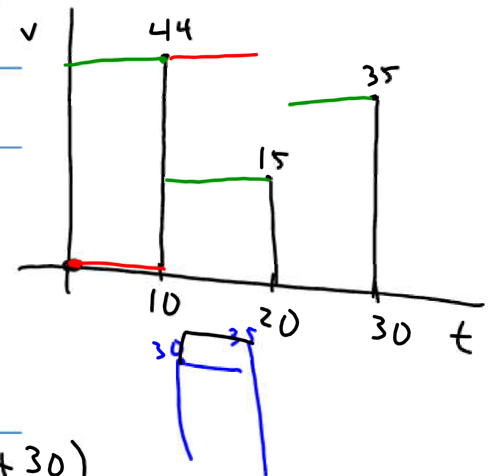
18. ***Length of a Road*** You and a companion are driving along a twisty stretch of dirt road in a car whose speedometer works but whose odometer (mileage counter) is broken. To find out how long this particular stretch of road is, you record the car's velocity at 10-sec intervals, with the results shown in the table below. (The velocity was converted from mi/h to ft/sec using 30 mi/h = 44 ft/sec.) Estimate the length of the road by averaging the LRAM and RRAM sums. 3665 ft

Time (sec)	Velocity (ft/sec)	Time (sec)	Velocity (ft/sec)
0	0	70	15
10	44	80	22
20	15	90	35
30	35	100	44
40	30	110	30
50	44	120	35
60	35		



LRAM: $10(0+44+15+\cdots+30)$

RRAM: $10(44+15+35+\cdots+35)$

$$\frac{\text{LRAM}+\text{RRAM}}{2} = 3665 \text{ ft}$$

Exercises 5–8 refer to the region R enclosed between the graph of the function $y = 2x - x^2$ and the x-axis for $0 \le x \le 2$.

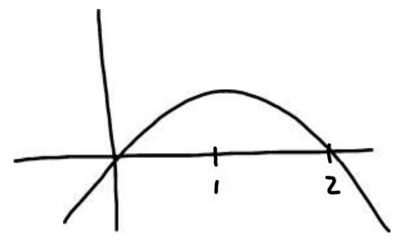
5. **(a)** Sketch the region R.

(b) Partition [0, 2] into 4 subintervals and show the four rectangles that LRAM uses to approximate the area of R. Compute the LRAM sum without a calculator.

6. Repeat Exercise 1(b) for RRAM and MRAM.

TRY:

1. $y = 2x - x^2$ on $[0, 2]$



a) Sketch region

b) Partition into 4 subintervals.

Find LRAM $= 0 + \frac{3}{8} + \frac{1}{2} + \frac{3}{8} = \frac{5}{4}$

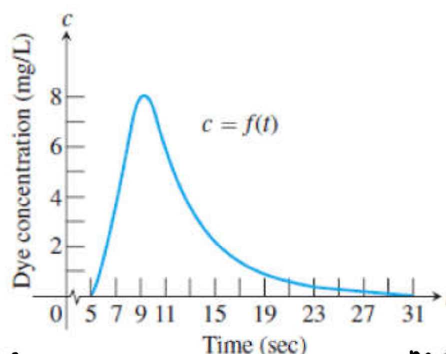
Cardiac Output

The number of liters of blood your heart pumps in a fixed time interval is called your *cardiac output*. For a person at rest, the rate might be 5 or 6 liters per minute. During strenuous exercise the rate might be as high as 30 liters per minute. It might also be altered significantly by disease. How can a physician measure a patient's cardiac output without interrupting the flow of blood?

One technique is to inject a dye into a main vein near the heart. The dye is drawn into the right side of the heart and pumped through the lungs and out the left side of the heart into the aorta, where its concentration can be measured every few seconds as the blood flows past. The data in Table 5.2 and the plot in Figure 5.10 (obtained from the data) show the response of a healthy, resting patient to an injection of 5.6 mg of dye.

Table 5.2 Dye Concentration Data

Seconds after Injection t	Dye Concentration (adjusted for recirculation) c
5	0
7	3.8
9	8.0
11	6.1
13	3.6
15	2.3
17	1.45
19	0.91
21	0.57
23	0.36
25	0.23
27	0.14
29	0.09
31	0

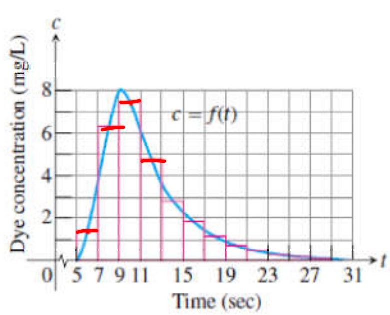


Know: 5.6 mg of dye
Want: L blood/min

area under curve: $\frac{\text{mg dye}}{\text{L}} \cdot \text{sec}$

$$\frac{\text{mg of dye}}{\frac{\text{mg dye}}{\text{L blood}} \cdot \text{sec}} = \text{mg dye} \cdot \frac{\text{L blood} \cdot \text{s}}{\text{mg dye} \cdot \text{sec}}$$

$$= \frac{\text{L blood}}{\text{sec}} \cdot \frac{60\text{ sec}}{\text{min}} = \text{L blood/min}$$



$$\text{Area} \approx f(6)\cdot 2 + f(8)\cdot 2 + f(10)\cdot 2 + \cdots + f(28)\cdot 2$$
$$\approx 2\cdot(1.4 + 6.3 + 7.5 + 4.8 + 2.8 + 1.9 + 1.1 + 0.7 + 0.5 + 0.3 + 0.2 + 0.1)$$
$$= 2\cdot(27.6) = 55.2\ (\text{mg/L})\cdot\text{sec}.$$

$$\frac{5.6\text{ mg}}{55.2\text{ mg}\cdot\text{sec/L}}\cdot\frac{60\text{ sec}}{1\text{ min}} \approx 6.09\text{ L/min}.$$

HW: p270 #5, 6, 15, 17, 19, 28
calculator!